FAR BEYOND

MAT122

Integration by Parts (IBP)

Stony Brook University

Integration by Parts

Often multiplied functions being integrated don't contain a useful derivative to use u-substitution.

Another integration tool is **Integration by Parts (IBP)**.

One function will be labeled u and the other function will be considered a derivative (not of u).

ex. $\int xe^x dx$

$$\int \underbrace{x}_{u} \cdot \underbrace{e^x dx}_{dv}$$

Build a chart:

$u = x$	$dv = e^x dx$
$du = 1 \cdot dx = dx$	$v = e^x$

pick the one that simplifies as a derivative to be u

the rest becomes dv

Integration by Parts Formula:

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= xe^x - \underbrace{\int e^x dx}$$

now the integral is easy to take

$$= \boxed{xe^x - e^x + C}$$

Integration by Parts – Example #1

$$\int u \cdot dv = uv - \int v \cdot du$$

ex. $\int xe^{3x}\,dx$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= \boxed{\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C}$$

Integration by Parts – Example #2

ex. $\int x^3 \ln x \, dx$

$$= \boxed{\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C}$$

IBP with One Function

IBP can be used for integrating a single function when it *has* a derivative but is *not* a derivative of the function.

ex. $\int \ln x \, dx$

Build a chart:

$$= \boxed{x \ln x - x + C}$$

$$\boxed{\int u dv = uv - \int v du}$$

ex. $\int_2^3 \ln x \, dx$

$$= \boxed{3\ln 3 - 2\ln 2 - 1}$$

More Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Sometimes, IBP must be taken multiple times:

ex. $\int x^2 e^x \, dx$

$$= \boxed{x^2 e^x - 2(xe^x - e^x) + C}$$