

**FAR
BEYOND**

MAT122

Integration by Parts (IBP)



Stony Brook University

Integration by Parts

Often multiplied functions being integrated don't contain a useful derivative to use u -substitution.

Another integration tool is **Integration by Parts (IBP)**.

One function will be labeled u and the other function will be considered a derivative (not of u).

ex. $\int x e^x dx$

pick the one that simplifies
as a derivative to be u

the rest becomes dv

$$\int \underbrace{x}_u \cdot \underbrace{e^x dx}_{dv}$$

Build a chart:

$$u = x \\ du = 1 \cdot dx = dx$$

$$dv = e^x dx \\ v = e^x$$

Integration by Parts Formula:

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= x e^x - \underbrace{\int e^x dx}_{\text{now the integral is easy to take}}$$

$$= \boxed{x e^x - e^x + C}$$

Integration by Parts – Example #1

ex. $\int x e^{3x} dx$

$$\int u \cdot dv = uv - \int v \cdot du$$

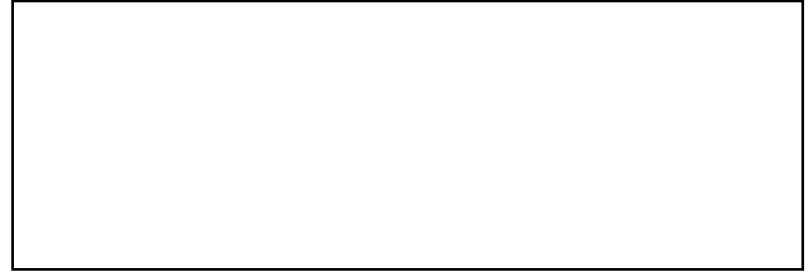


$$\int u \cdot dv = uv - \int v \cdot du$$

$$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C$$

Integration by Parts – Example #2

ex. $\int x^3 \ln x \, dx$



$$= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C$$

IBP with One Function

IBP can be used for integrating a single function when it *has* a derivative but is *not* a derivative of the function.

ex. $\int \ln x \, dx$

Build a chart:

$$= \boxed{x \ln x - x + C}$$

ex. $\int_2^3 \ln x \, dx$

$$= \boxed{3 \ln 3 - 2 \ln 2 - 1}$$

$$\int u \, dv = uv - \int v \, du$$

More Integration by Parts

Sometimes, IBP must be taken multiple times:

ex. $\int x^2 e^x dx$

$$\int u dv = uv - \int v du$$

$$= x^2 e^x - 2(xe^x - e^x) + C$$